Damage Localization via Transmissibility Power Mode Shape

Yun Lai ZHOU*, Ricardo PERERA*
* Technical University of Madrid
Jose Gutierrez Abascal 2, 28006 Madrid
[a] yunlai.zhou@alumnos.upm.es

Abstract. Damage localization plays an increasingly important role in structural health monitoring of numerous engineering applications. In order to achieve this goal, many methods have been developed relating to the dynamic features between the healthy and damaged statuses. Especially the variation of natural frequencies and mode shapes data have been undertaken investigation. Transmissibility has been of continuing interest to the scientific communities in recent years and became a well studied topic due to its property that only depends on the output signal. In this paper a method of damage localization by using transmissibility power mode shape is proposed. Above all, transmissibility is implemented in the frequency domain from the output data measurements after choosing a reference point. Then, average transmissibility to different reference points would be calculated. Consequently, transmissibility power mode shape is constructed via the transmissibility as to a particular point apart from the boundary points. Henceforth, modal parameters and damage localization indices are generalized to analyze the structural properties and identify as well as localize the damages under different loading conditions. Finally, the Gaussian white noise is added into the numerical simulation to illustrate the feasibility of the proposed damage localization parameters in engineering applications. Simulation results reveal the excellent performances of the method not only able to detect and localize damage but also to assess different damage levels.

1. Introduction

The worldwide civil infrastructures have greatly brought convenience to the human beings while their aging problems are increasingly exposing in the recent decades. Many researchers have devoted to studying the structural health monitoring (SHM) prognostic health management (PHM): damage identification and localization, structural remaining performance time as well as extension of the performance time.

In the beginning, due to the fact that the structural damage will cause change in structural parameters involving the mass, damping and stiffness matrices which will inherently result in the changes of dynamic properties: a reduction in natural frequencies, an increase in modal damping, and a change to the mode shape, researchers have developed vibration-based techniques to estimate the structural health. Pandey et al. proposed the mode shape curvature method and the flexibility matrix method [1,2]. Stubbs et al. proposed the strain energy method [3]. Fang and Perera [4] advised using power mode shape for early damage detection. However, vibration-based structural health monitoring methods normally need to extract vibration characteristics from operational vibration measurements.

Transmissibility is a relatively new research direction which only depends on the vibration response. Due to its own characteristic, researchers have developed some methods based on this new conception. Almost twenty years ago, Q. Chen et al [5] firstly proposed transmissibility functions as potential features for damage detection. Afterwards, more transmissibility based methods are developed by more and more researchers [6,7,8,9,10].

This paper is organized as: firstly the transmissibility and its own characteristics are introduced and the transmissibility power mode shape is established. Then the related damage localization indices are shown how they could be derived from the transmissibility power mode shape. Afterwards, a numerical example is given will be addressed so as to check the feasibility of potential use in real engineering. Finally the conclusions will be drawn.

2. Method

2.1 Transmissibility

Transmissibility measurement is an increasing widely used technique, and is very suitable for operational dynamic analysis in structural health monitoring. Here, the power spectrum density transmissibility (PSDT) function is defined as the ratio of two responses spectra by assuming a single force applied in an input degree of freedom. Normally, in real life several operational forces or even more complicated forces are exciting the structure, as this happens, it would be quite more sophisticated in calculation. In order to avoid this problem here using a reference response signal instead of an excitation signal to estimate the power spectrum density transmissibility.

As to power spectrum density, the transmissibility between the outputs $y_i(t)$ and $y_j(t)$ with reference to the output $y_p(t)$ is defined as the ratio between the power spectral densities responses $X_i^p(\omega)$ and $X_j^p(\omega)$:

$$T_{ij}^p(\omega) = \frac{X_i^p(\omega)}{X_j^p(\omega)} \tag{1}$$

where ω is the frequency.

Actually, in several ways transmissibility could be derived, one of the most common ways is the use of cross- and auto-power functions G :

$$\left|T_{ij}^p(\omega)\right| = \sqrt{\frac{X_i(\omega)X_p(\omega)}{X_j(\omega)X_p(\omega)}} = \sqrt{\frac{G_{Y_iY_p}(\omega)}{G_{Y_jY_p}(\omega)}} \tag{2}$$

Furthermore, as has been proved [11, 12], when the Laplace variable approaches system's vth pole, denoted by λ_v, the following equation is verified as:

$$\lim_{s\to\lambda_v} T_{ij}^p(s) = \frac{\phi_{iv}}{\phi_{jv}} \tag{3}$$

Therefore, if to choose two different reference points, k and l, the subtraction of the two PSDTs satisfies

$$\lim_{s\to\lambda_v}(T_{ij}^k(s) - T_{ij}^l(s)) = \frac{\phi_{iv}}{\phi_{jv}} - \frac{\phi_{iv}}{\phi_{jv}} = 0 \tag{4}$$

This means that the system's poles are zeros of the rational function:

$$\Delta T_{ij}^{kl}(s) = T_{ij}^k(s) - T_{ij}^l(s) \tag{5}$$

And its inverse which called inverse transmissibility subtraction function (ITSF) is as:

$$\Delta^{-1}T_{ij}^{kl}(s) = \frac{1}{\Delta T_{ij}^{kl}(s)} = \frac{1}{T_{ij}^{k}(s) - T_{ij}^{l}(s)} \tag{6}$$

The normalized inverse transmissibility subtraction function (NITSF), and the average normalized inverse transmissibility subtraction function (ANITSF) are as:

$$NITSF_n = \frac{\Delta_n^{-1}T_{ij}^{kl} - \Delta_{\min}^{-1}T_{ij}^{kl}}{\Delta_{\max}^{-1}T_{ij}^{kl} - \Delta_{\min}^{-1}T_{ij}^{kl}} \tag{7}$$

$$ANITSF = \frac{1}{N}\sum_{n=1}^{N} NITSF_n \tag{8}$$

The above theoretical results conclude that transmissibility is feasible to establish a rational function $\Delta^{-1}T_{ij}^{kl}(s)$, with poles equal to the system's poles.

2.2 Damage localization indices

As shown in the previous section, The transmissibility at the system poles are coincident with values of the mode shape ratios, i.e. the values of the T_{ij} at the system poles are directly related to the scalar operational mode-shape values ϕ_{iv} and ϕ_{jv}. Therefore, once the resonant frequencies are identified by using ANITSF, it is also possible to identify the operational mode shape vectors from different PSDTs. By choosing a fixed reference DOF j and giving ϕ_{jv} a normalized value of unit, the full unscaled mode-shape (operational deflection) vector $\left(\phi_{1v}, \phi_{2v}, \cdots, 1, \cdots, \phi_{Kv}\right)$ (K is the number of measured output DOFs) can be constructed from the PSDT vector $\left(T_{1j}, T_{2j}, \cdots, 1, \cdots, T_{Kj}\right)$. Then, by analogy with the concept of power mode shape presented in [13], a new concept of transmissibility power mode shape (TPMS) might be defined from the PSDT in the following way:

$$TPMS_i^{v} = \int_{f_{v1}}^{f_{v2}} T_{ij}(f)df \tag{9}$$

where $TPMS_i^{v}$ is the ith component of the υth transmissibility power mode shape and $f_{\upsilon 2} - f_{\upsilon 1}$ is the integrated frequency bandwidth for the υth TPMS.

By assembling $TPMS_i^{v}$ for all the measured points considered in the structure, a υth TPMS vector is generated:

$$\left\{TPMS^{v}\right\} = \left(TPMS_1^{v}, TPMS_2^{v}, \cdots, 1, \cdots, TPMS_K^{v}\right) \tag{10}$$

The same procedure should be repeated for each TPMS by choosing the appropriate bandwidth affecting each system's pole υ. In this way, any of the damage criteria based on mode shapes might be extended to include the transmissibility power mode shapes.

As to identify the structural damages, based on mode shape, the curvature is another parameter to detect and localize the damage which is defined as the second derivative of mode shape.

$$\phi''_{i,j} = \frac{\phi_{i,j+1} - \phi_{i,j} + \phi_{i,j-1}}{l^2} \tag{11}$$

The location of the existing damage is estimated by the absolute change in mode shape curvature of healthy and unhealthy structure and expressed as

$$MSC_i = \sum_j \left|\phi''^{*}_{ij} - \phi''_{ij}\right| \tag{12}$$

As used here, the transmissibility power mode shape curvature change will be showed as

$$\Delta TPMSC_j = \left|TPMSC^{*}_j - TPMSC_j\right| \tag{13}$$

In order to show the real state of increase and decrease, in this paper the real subtraction value other than the absolute value will be used.

And a damage index method constructed by Stubbs, Kim and Farrar [14], which uses the pre-damage and post-damage modal curvature, has been proved to be feasible for localizing the damages. It is defined as

$$\beta_{ij} = \frac{\left(\int_a^b [\phi''^{*}_j(x)]^2 dx + \int_0^L [\phi''^{*}_j(x)]^2 dx\right) \quad \int_0^L [\phi''^{*}_j(x)]^2 dx)}{\left(\int_a^b [\phi''_j(x)]^2 dx + \int_0^L [\phi''_j(x)]^2 dx\right) \quad \int_0^L [\phi''_j(x)]^2 dx)} \tag{14}$$

As to the discrete system, a more common damage localization index is indicated as

$$\beta_{ij} = \frac{(\phi''^{*2}_{ij} + \sum_{i=1}^{N} \phi''^{*2}_{ij}) \sum_{i=1}^{N} \phi''^{2}_{ij}}{(\phi''^{2}_{ij} + \sum_{i=1}^{N} \phi''^{2}_{ij}) \sum_{i=1}^{N} \phi''^{*2}_{ij}} \tag{15}$$

Considering the transmissibility power mode shape (TPMS) here, as to all the nodes and jth mode in the calculation, damage index in each element

$$DI_{ij} = \frac{(TPMS''^{*2}_{ij} + \sum_{i=1}^{N} TPMS''^{*2}_{ij}) \sum_{i=1}^{N} TPMS''^{2}_{ij}}{(TPMS''^{2}_{ij} + \sum_{i=1}^{N} TPMS''^{2}_{ij}) \sum_{i=1}^{N} TPMS''^{*2}_{ij}} \tag{16}$$

And to more modes, the damage index to the ith node is

$$DI_{i} = \sum_{j=1}^{Nm} \frac{(TPMS''^{*2}_{ij} + \sum_{i=1}^{N} TPMS''^{*2}_{ij}) \sum_{i=1}^{N} TPMS''^{2}_{ij}}{(TPMS''^{2}_{ij} + \sum_{i=1}^{N} TPMS''^{2}_{ij}) \sum_{i=1}^{N} TPMS''^{*2}_{ij}} \tag{17}$$

Then, to one node for all the measured modes, a normalized damage index is described as follows:

$$NDI_i = \frac{DI_i - \min(DI_i)}{\max(DI_i) - \min(DI_i)} \tag{18}$$

Actually,

$$\begin{aligned} DI_{i} &= \sum_{j=1}^{Nm} \frac{(TPMS''^{*2}_{ij} + \sum_{i=1}^{N} TPMS''^{*2}_{ij}) \sum_{i=1}^{N} TPMS''^{2}_{ij}}{(TPMS''^{2}_{ij} + \sum_{i=1}^{N} TPMS''^{2}_{ij}) \sum_{i=1}^{N} TPMS''^{*2}_{ij}} \\ &= \frac{\sum_{j=1}^{Nm} TPMS''^{*2}_{ij} + \sum_{j=1}^{Nm}\sum_{i=1}^{N} TPMS''^{*2}_{ij}}{\sum_{j=1}^{Nm} TPMS''^{2}_{ij} + \sum_{j=1}^{Nm}\sum_{i=1}^{N} TPMS''^{2}_{ij}} \times \frac{\sum_{j=1}^{Nm}\sum_{i=1}^{N} TPMS''^{2}_{ij}}{\sum_{j=1}^{Nm}\sum_{i=1}^{N} TPMS''^{*2}_{ij}} \end{aligned} \tag{19}$$

If assuming

$$\begin{aligned} \sum_{j=1}^{Nm}\sum_{i=1}^{N} TPMS''^{2}_{ij} &= C1 \\ \sum_{j=1}^{Nm}\sum_{i=1}^{N} TPMS''^{*2}_{ij} &= C2 \end{aligned} \tag{20}$$

To each state of the structure, C1 and C2 are constant.

Then

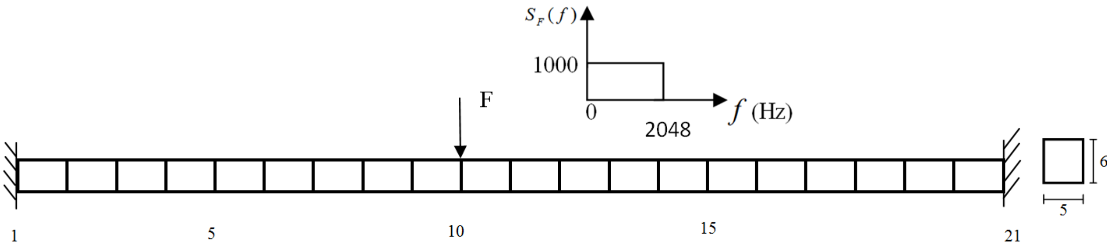
$$DI_i = \frac{\sum_{j=1}^{Nm} TPMS_{ij}''^{*2} + C2}{\sum_{j=1}^{Nm} TPMS_{ij}''^{2} + C1} \times \frac{C1}{C2} \tag{21}$$

Thus, define a new damage index SDI.

$$SDI_{ij} = \frac{TPMS_{ij}''^{*2}}{TPMS_{ij}''^{2}} \tag{22}$$

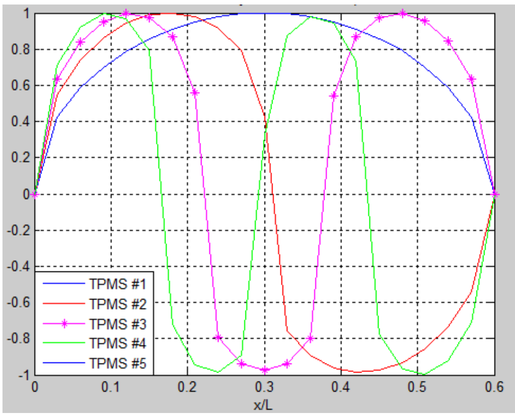
3. Case studies

Numerical simulation is carried out with a two sided fixed beam with length 0.6 m, which was adopted to examine the performance of the proposed damage detection indices. A schematic diagram of this beam with its geometric dimensions and material properties is shown in Pic. 1. Different simulations were carried out by considering a mesh of 20 beam elements. The beam was assumed to be lightly damped with a constant damping ratio of 0.5%. And the loading point is node 10 with a vertical force of 1000 from 0 to 2047 Hz.



Pic. 1. Two sided fixed beam model

The transmissibility power mode shapes are shown in Pic. 2 when no stiffness reduction and no noise are introduced to the simulation.

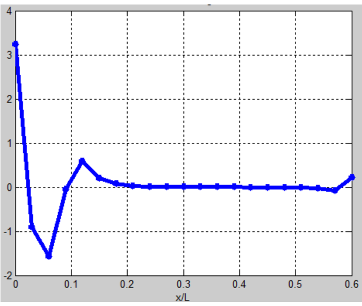


Pic. 2. Transmissibility power mode shapes

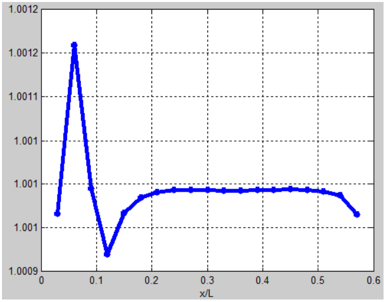
As shown in Pic. 2, the transmissibility power mode shapes are quite similar to the mode shapes. Here the damage localization indices will be estimated based on these transmissibility power mode shapes.

3.1 Noise free scenarios

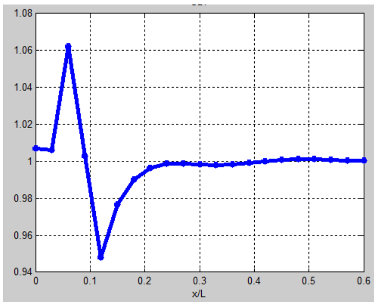
A stiffness reduction of 10% is firstly introduced as the single damage to the element 3 (node 3-4). The transmissibility power mode shape curvature change (TPMSC), DI index and SDI index are shown in Pic. 3 to Pic. 5.



Pic. 3. TPMSC change

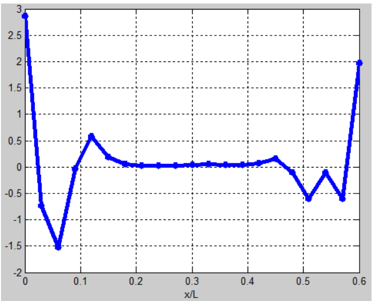


Pic. 4. DI

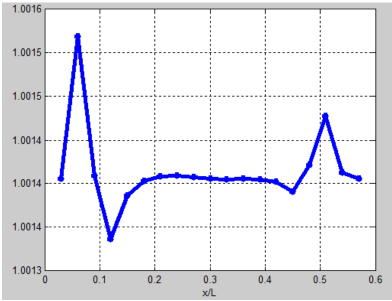


Pic. 5. SDI

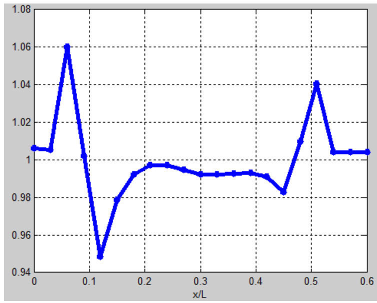
Then a stiffness reduction of 10% is introduced to both the element 3 (node 3-4) and 17 (node 17-18). The transmissibility power mode shape curvature change (TPMSC), DI index and SDI index are shown in Pic. 6 to Pic. 8.



Pic. 6. TPMSC change



Pic. 7. DI

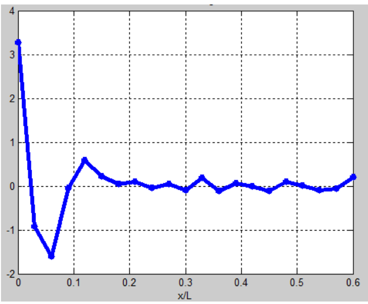


Pic. 8. SDI

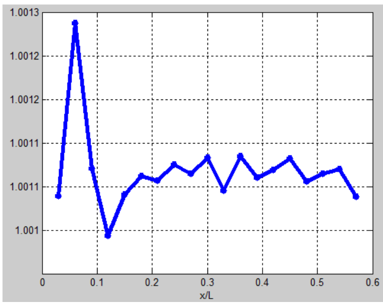
Pic. 3, Pic. 4 and Pic. 5 show that the transmissibility power mode shape curvature change (TPMSC), DI index and SDI index can detect and localize the damaged elements while the SDI has lower performance in those health elements compared to the TPMSC and DI, which means the SDI has more clear localization results. And to the multiple damages, all the three indices can also detect and localize damages. Pic. 7 and Pic. 8 show almost the same clear localization results which mean they successfully localize the damaged elements.

3.2 Noisy scenarios

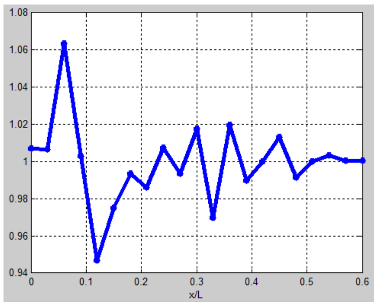
The damaged elements and stiffness reduction are the same as described in the noise free scenarios. Here 10% noise of normally distributed with zero mean value as well as a standard deviation is introduced in to the simulation model. Henceforth, the TPMSC change, DI index and SDI index results are shown in Pic. 9 to Pic. 14.



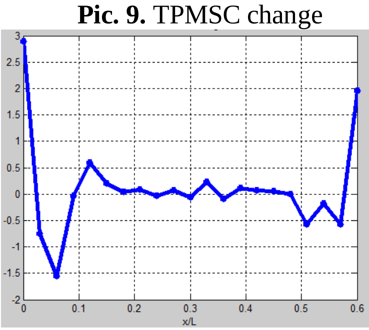
Pic. 9. TPMSC change



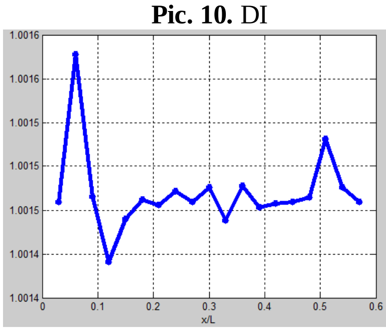
Pic. 10. DI



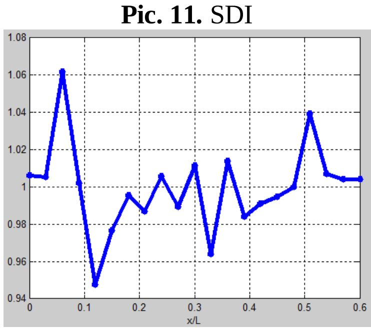
Pic. 11. SDI



Pic. 12. TPMSC change



Pic. 13. DI



Pic. 14. SDI

Pic. 9, Pic. 10 and Pic. 11 show that in the single damage situation with 10% of noise, the TPMSC change, DI and SDI can still suitably localize the damaged elements. And Pic. 12, Pic. 13 and Pic. 14 show that as to the multiple damages situation, DI and SDI show better performance than TPMSC change in detecting and localizing the damages. When compared with the noise free scenarios, it is easy to find that the noise causes some unexpected peaks in the final results which influenced the damage localization results. Those Pic.s also show that when noise is introduced into the structure, the DI works better than the SDI and TPMSC change.

4. Conclusions

The objective of this study is to implement a new method based on the transmissibility power mode shape (TPMS). The proposed method upon the use of transmissibility power mode shape (TPMS) shows great promising future in damage localization. Above, the proposed method only depends on the output response signals and has no demands for FRF, IRF and so on. Secondly, the proposed method has a good tolerance for the noise which is a quite common thing in the real engineering environments. Lastly, the method can be also used to localize the damages for complex structures. One thing should be paid attention is that the loading point should be well chosen in order to achieve a better transmissibility power mode shape (TPMS) which can be fulfilled by the experience.

Acknowledgments

The writers acknowledge the support for the work reported in this paper from the Spanish Ministry of Science and Innovation (project BIA2010-20234-C03-01). Financial support for the CSC research fellowship given to Yunlai Zhou is also acknowledged.

References

[1] A.K. Pandey, M. Biswas, M.M. Samman, Damage detection from changes in curvature mode shapes, Journal of Sound and Vibration 145 (2) (1991) 321–332.

[2] A.K. Pandey, M. Biswas, Damage detection in structures using changes in flexibility, Journal of Sound and Vibration 169 (1) (1994) 3–17.

[3] N. Stubbs, J.T. Kim, K. Topole, An efficient and robust algorithm for damage localization in offshore platforms, Proceedings of the ASCE 10th Structures Congress , 1992, pp. 543–546.

[4] Sheng-En Fang, Ricardo Perera, Power mode shape for early damage detection in linear structures, Journal of Sound and Vibration 324 (2009) 40–56.

[5] Q. Chen, Y.W. Chan, K. Worden, G.R. Tomlinson, Structural fault detection using neural networks trained on transmissibility functions, in: Proceedings of the International Conference on Vibration Engineering, Beijing, China, 1994, pp 456–646.

[6] Yunlai Zhou, Ricardo Perera, Enrique Sevillano. Damage identification from power spectrum density transmissibility, 6th European Workshop on Structural Health Monitoring and 1th European Conference of the Prognostics and Health Management (PHM) Society; Dresden, July 2012

[7] Yang, W.J., Ren, W.X., Operational Modal Parameter Identification from Power Spectrum Density Transmissibility, Computer-Aided Civil and Infrastructure Engineering, 27(2012), 202-217.

[8] Devriendt, C., De Sitter, G., Guillaume, P., An operational modal analysis approach based on parametrically identified multivariable transmissibilities, Mechanical Systems and Signal Processing, 24(2010), 1250-1259.

[9] Lang, Z.Q., Park, G., Farrar, C.R., Todd, M.D., et al, Transmissibility of non-linear output frequency response functions with application in detection and location of damage in MDOF structural systems, International Journal of Non-Linear Mechanics, 46 (2011), 841-853.

[10] Maia, N.M.M., Almeida, R.A.B., Urgueira, A.P.V., Sampaio, R.P.C., Damage detection and quantification using transmissibility, Mechanical Systems and Signal Processing, 25 (2011), 2475-2483.

[11] Devriendt, C. & Guillaume, P. (2007), The use of transmissibility measurements in output-only modal analysis, Mechanical Systems and Signal Processing, 21(7), 2689-2696.
[12] Devriendt, C., Guillaume, P. (2008), Identification of modal parameters from transmissibility measurements, Journal of Sound and Vibration, 314, 343-356.
[13] Allemang, R.J., Brown, D.L. (1982), A correlation for modal vector analysis, Proceedings of 1st International Modal Analysis Conference, 110-116.
[14] N. Stubbs, J. T. Kim and C. R. Farrar (1995), Field verification of a non-destructive damage localization and severity estimation algorithm, Proceedings of the XIII International Modal Analysis Conference, Nashville, USA.