

Estimating Probability of Detection Curves Related to Eddy Current Sender – Receiver Probes

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Abstract. Sender - receiver probes are commonly used in eddy current procedures for crack detection. Such probes can give a non-linear signal response as a function of the crack size. Classical methods used for analysis in probability of detection (POD) assessments require that the signal response versus crack size can be predicted with a linear relation, which also must show a constant variance of the collected data points. One approach which can be used to overcome these limitations is to use a mathematical model of the eddy current probe – flaw interaction and use this to estimate the detection probability at specific crack sizes. This method is applied within this work, using the finite element method for the eddy current signal response predictions. In order to manage a large number of calculations at several crack sizes we propose the use of a meta-model approach. A linear meta-model is created at different crack sizes and then used for POD estimation. The number of signal responses above the detection level at specific crack sizes is then used to estimate the POD, a method which does not require any particular relation between signal response and crack size. The meta-model enables a large number of stochastic computations to be carried out in order to estimate the signal response distribution for a specific crack size. We conclude that modelling is a vital part of procedure capability estimations of eddy current procedures based on sender – receiver probe characteristics and can be used for procedure understanding, improvements and aid in experimental verifications.

1. Introduction

Eddy current inspection procedures are often used for critical non-destructive testing (NDT) and evaluation (NDE) in the aerospace industry. Work on method capability through estimation of the probability of detection (POD) curve is important due to the large variation in method capability depending on equipment, procedures, object geometry and defect characteristics. Eddy current sensors can operate in different modes such as absolute and differential measuring the impedance response compared to a fixed state or as the difference between elements of the sensor. One mode that is gaining more interest is the sender – receiver configuration. This can be seen in array systems utilizing many coils often to enhance the inspection coverage in individual scans or adapted to a specific geometry. The sender – receiver configuration is also highly interesting for systems utilizing a different sensing technology apart from the receiving coil, such as Hall sensors,



the quantum interference device (SQUID), sensors based on the giant magnetoresistive effect (GMR) [1] or other approaches e.g. [2].

The objective here is to study how mathematical modelling can be used to aid in the understanding and estimation of POD for sender – receiver probes. Here, we are studying sender – receiver sensors based on two coils placed side by side along the scan direction for detection of small surface breaking cracks. The probe configuration is non-axial and is preferable if appearing cracks have a known orientation, in this case parallel to the scan direction. The configuration will be sensitive to the coil distance C_D , see Figure 2, between the sender and receiver. This is exemplified in Figure 1 presenting experimental data from a sender – receiver probe scanned over two cracks with different surface length *a*. The smaller crack is giving significantly larger signal response. It can be concluded that we might not expect an increase in signal response for an increase in crack size. The length scales in the figure is related to the coil distance C_D which we suggest to use, as a scaling factor, in order to be as general as possible.



Figure 1. Experimental impedance data from a sender – receiver probe scanned over two cracks with different surface length a. C_D is the distance between the centres of the sender and receiver coils.

When a sender – receiver system is studied in terms of reliability and POD estimation the nonlinear signal response related as a function of crack size will introduce a problem using conventional principles [3]. In order to estimate a parametric POD curve it is assumed that a linear relation between signal response and crack size exists. The idea is also that the POD will monotonically increase from zero to 100 %. New approaches using Bayesian statistics is demonstrated e.g. under the condition that the POD will hit a limit below 100 % detection probability [4]. However, the sender - receiver system addressed here must be assumed to, under certain circumstances, also have a lower POD at larger defect sizes. We suggest to increase the knowledge of such system and to estimate the POD by the use of model based methods. A mathematical model can be used to achieve a non-parametric POD curve [5]. A similar approach could be attempted using experiments but requires a large amount of input data. A non-parametric estimation will handle nonlinear and non-monotonically increasing signal responses to crack sizes as well as the possibility that signal response distribution will vary between different crack size regions.

2. Mathematical Model

The eddy current procedure using a sender – receiver probe is here studied within a mathematical model. The model is based on the work carried out in [6] and is based on numerical implementation using the finite element method (FEM). The inspection is

assumed to be carried out on a flat surface Titanium alloy Ti-6Al-4V. The probe is including a sending coil with an inner radius of 0.3 mm, an outer radius of 0.4 mm, a length of 1 mm and with an applied current at a frequency of 3 MHz. The receiver is placed beside the sender at a distance, C_{D_i} of 1 mm, see Figure 2. The receiver is assumed to be thin with a radius of 0.3 mm. The impedance response curve in the figure shows a similar setup as previously presented in figure 1, even though the dimensions of the sensors are different. The cracks included in the model are assumed to be half-circular or elliptic and open to the surface.



Figure 2. Modelled signal response from a sender – receiver probe scanned over two crack with different surface length *a*. The smaller crack is giving significantly larger signal response.

The impedance phase is adjusted such that the Z_y response is maximized for the calibration configuration. The calibration used is a rectangular notch, approximately 100 μ m wide, with a length and depth equal to C_D and $C_D/2$ respectively.

2.1 Nominal signal response predictions

The calculated nominal signal response $\hat{a} = |Z_{y(max)}-Z_{y(min)}|$ is presented as a function of crack size, *a*, in Figure 3. The nominal parameter values of the procedure are the centre values of Table 1. Figure 1 includes the signal response predictions for rectangular notches as well as half circular cracks with three different widths. The crack width has a significant impact on the magnitude of the signal response but not on the function shape. Here we use crack widths around 100 µm in order to reduce the number of degrees of freedom needed in the FE model, which grows for small crack width especially regarding larger crack sizes.



Figure 3. Calculated, nominal, signal responses as a function of crack size, *a*.

The signal response, corresponding to nominal parameter values, of the sender - receiver probe confirms that it is strongly dependent on the crack size and has an optimum, which here is represented around 1.75 mm crack length, also corresponding to $a/C_D = 1.75$.

2.2 Significant Variables of the Procedure

We consider a flat surface inspection procedure. The occurring cracks are expected in a direction parallel to the scan but are allowed to deviate 10° from the ideal orientation. The process variables that we assume to be uncertain during the procedure are listed in Table 1. The distributions of the uncertain procedure parameters are assumed to be uniform between the minimum and maximum value presented in the table.

Variable Description		Minimum	Maximum	
α	Crack orientation relative scan	-10°	10°	
β	Crack orientation relative surface normal vector	-10°	10°	
σ_0	Bulk conductivity	522 000 S/m	638 000 S/m	
a_D	Crack depth	$0.4 \cdot a$	$0.6 \cdot a$	
a_W	Crack width	80 µm	120 µm	
y ₀	Scan position relative crack centre	-0.2 mm	0.2 mm	
Z ₀	Lift - off	80 µm	120 µm	

Table 1. Procedure uncertainties, all parameters are assumed to be uniformly distributed between maximum and minimum values. The crack is elliptical with a surface length *a*.

In order to estimate a nonparametric POD curve from modelled signal response predictions, we need a large number of computations. In order to have efficient calculation we suggest to create a meta-model based on the FEM calculations. The uncertain variables vary around the nominal value and we assume that this variation can be approximated as linear. However, we can conclude that the signal response is non-linear as a function of crack size, which is clear from Figure 3. Linear regression models are therefore built at individual crack sizes, a = 0.5, 0.75, 1, ..., 4.5 mm. The idea was to first attempt to reduce the number of significant variables that must be represented as uncertain and coupled to a distribution. This could simplify the meta-model construction or reduce the number of input data. A full factorial test was applied for two crack sizes in an attempt to reduce the number of important variables. The selected crack sizes represent different size regimes, the resulting effects are significantly different, which can be concluded from Table 2. We therefore include all seven variables as uniformly distributed for the POD estimation, even if the result suggests reducing β , as this variable has the small impact on both crack sizes.

Table 2. Normalized effects from a full factorial test at two crack sizes.

Crack Length	Effect size in decreasing order of magnitude (largest 7)							
1.5 mm	z ₀ (-0.253)	$a_{\rm D}(0.170)$	y ₀ (-0.161)	$\sigma_0(0.088)$	$a_W(0.073)$	$\beta \cdot y_0 (0.061)$	α (-0.04)	
3.5 mm	z ₀ (-1.96)	$\alpha \cdot y_0 (0.156)$	$\sigma_0(0.075)$	α (-0.071)	$a_W(0.048)$	$\beta \cdot y_0 (0.022)$	α·β (-0.02)	

2.3 Meta-Model Construction

The meta-model is represented as a linear regression model including all variables in Table 1. The model fitting is carried out in the least square sense based on 20 signal responses at each crack size (0.5, 0.75, 1.0, ..., 4.5 mm) calculated using the FEM. The input data points

were selected to fill the design space under the condition that new points are added sequentially where the distance to the existing points is maximized. Figure 4 is showing signal prediction using the meta-model where the input parameters are sampled from uniform distributions according to Table 1. The result of the usage of a meta-model instead of directly compute the signal responses with FEM is that we increased the computation efficiency to be around $10^4 - 10^5$ times faster.



Figure 4. Signal response predictions using the meta-model. The uncertain variables of the procedure are sampled from uniform distributions according to Table 1.

The meta-model was used to calculate 2000 signal responses at each crack size. A small number of these points are presented in Figure 4. The signal responses occur more often below the nominal response curve as this represents an optimal setting for α , β and y_0 . The other parameters all have their mean value at the nominal parameter setting. Latin hypercube sampling was used in order to select representative sampling data from the distributions of the uncertain procedure parameters.

3. Results

The POD curve is presented in Figure 5, calculated using hit/miss principles. At each crack size, a, we base the result from 2000 signal responses.



Figure 5. Non-parametric POD curve estimated from modelled data.

Figure 5 is showing a strong dependence between the applied rejection level and the estimated POD curve. The POD curve does also correlate to the optimized signal response at specific crack size section.

4. Conclusions

The non-axial sender – receiver probes show a complex signal response at different crack sizes. Procedures based on such probe configurations can therefore be problematic to quantify in terms of capability and POD. We conclude that a mathematical model of the eddy current method and the sender – receiver probe can aid in the understanding of procedure capabilities of such probe configurations. An important key of the approach suggested here is shown to be the computation efficiency, gained though the use of a meta-model. The use of a mathematical model can

- be used in order to estimate non-parametric POD curves
- evaluate the procedure capability in relation to variations in procedure parameters,
- identify the significant variables in relation to e.g. crack size regimes,
- aid in the selection of crack sizes needed for experimental POD campaigns.

The mechanism and characteristics of the signal response of sender – receiver probes can be assessed in a mathematical model. We will continue this research directed both towards more complex geometries and to increase the understanding of NDE with respect to integrity and quality in a component life-cycle perspective by the use of mathematical models.

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